CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
MARINE ENGINEER OFFICER

EXAMINATIONS ADMINISTERED BY THE
SCOTTISH QUALIFICATIONS AUTHORITY
ON BEHALF OF THE
MARITIME AND COASTGUARD AGENCY

STCW 95 MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

Applied Heat
October 2018 Solution

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Note:-

This solution is for private circulation only. Not for sale.

Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise).We would be thankful to the reader, if they are brought to my attention at the following e-mail address: ***ulheyogesh@gmail.com***

Q.1

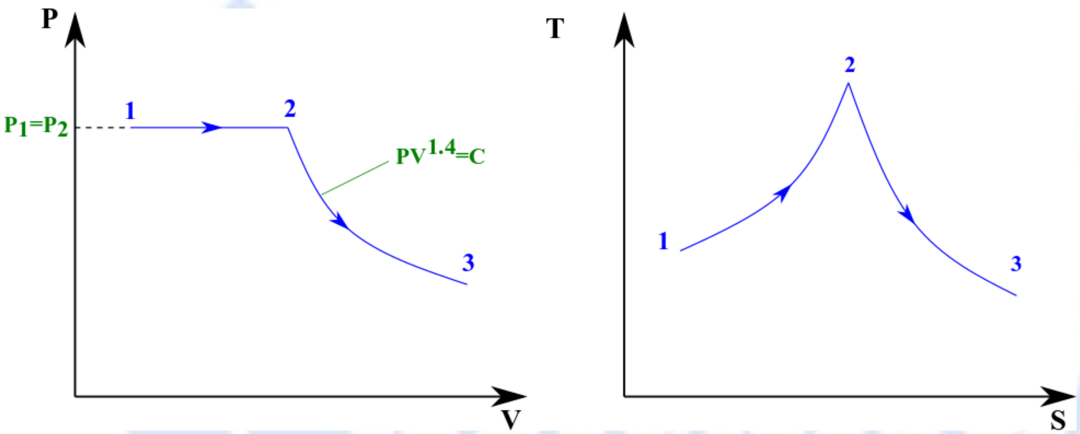
A perfect gas is heated at constant pressure and then expands reversibly according to the law $PV^{1.4}$=constant.
The initial pressure and temperature are 15 bar and 600°C respectively.
The final pressure is 1.0 bar and the final volume is 12 times the initial volume.
(a) Sketch the process on a Temperature-specific entropy diagram. (2)
(b) Calculate EACH of the following:

- **(i) the final temperature; (2)**
- **(ii) the temperature after the heating process; (2)**
- **(iii) the specific work transfer; (4)**
- **(iv) the net change in specific entropy for the polytropic process, (6)**

Note: for the gas c_v=5.179 kJ/kgK and R=2.078 kJ/kgK

Solution:

(a)



(b)

$$P_1 = 15\ bar = 1500\ kN/m^2 = P_2$$

$$T_1 = 600\,^oC = 873\ K$$

$$P_3 = 1\ bar = 100\ kN/m^2$$

$$V_3 = 12V_1$$

$$c_p = 5.179\ kJ/kgK, \qquad R = 2.078\ kJ/kgK$$

<u>For process 2-3</u>

$$P_2{V_2}^n = P_3{V_3}^n$$

$$\Rightarrow \left(\frac{V_2}{V_3}\right)^n = \frac{P_3}{P_2}$$

$$\Rightarrow \frac{V_2}{V_3} = \left(\frac{P_3}{P_2}\right)^{1/n} = \left(\frac{100}{1500}\right)^{1/1.4} = 0.1445$$

$$\Rightarrow V_2 = 0.1445V_3 = 0.1445 \times 12V_1 = 1.734V_1 \qquad \{\because V_3 = 12V_1$$

$$\Rightarrow \frac{V_2}{V_1} = 1.734$$

<u>For process 1-2</u>

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow T_2 = \frac{V_2}{V_1} \times T_1 = 1.734 \times 873 = 1513.782\ K$$

<u>For process 2-3</u>

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1}$$

$$\Rightarrow T_3 = T_2\left(\frac{V_2}{V_3}\right)^{n-1} = 1513.782 \times 0.1445^{0.4} = 698.25\ K$$

(b)(i)

$The\ final\ temperature\ = T_3 = 698.25\ K$

(b)(ii)

$The\ temperature\ after\ the\ heating\ process = T_2 = 1513.782\ K$

(b)(iii)

$$W_{total} = W_{1-2} + W_{2-3}$$

$$W_{1-2} = mR(T_2 - T_1) = 1 \times 2.078 \times (1513.782 - 873) = 1331.54\ kJ/kg$$

$$W_{2-3} = \frac{mR(T_2 - T_3)}{n-1} = \frac{1 \times 2.078 \times (1513.782 - 698.25)}{0.4} = 4236.69\ kJ/kg$$

$$W_{total} = 1331.54 + 4236.69 = 5568.23\ kJ/kg$$

Therefore, the specific work transfer is 5568.23 kJ/kg.

(b)(iv)

$$\Delta s_{2-3} = c_p \ln\left(\frac{T_3}{T_2}\right) - R\ln\left(\frac{P_3}{P_2}\right) = 5.179\ln\left(\frac{698.25}{1513.782}\right) - 2.078\ln\left(\frac{100}{1500}\right)$$

$$\Rightarrow \Delta s_{2-3} = 1.6198\ kJ/kgK$$

Therefore, the net change in specific entropy for polytropic process is 1.6198 kJ/kgK.

Q.2

In the open cycle gas turbine plant shown in Fig Q2, the HP turbine drives the compressor and the LP turbine drives the load.

Air enters the compressor at a pressure and temperature of 1.05 bar and 15°C respectively.

The combustion products enter the HP turbine at a pressure of 9.45 bar and temperature of 1027°C.

The gas entering the LP turbine is at a pressure of 3.23 bar and leaves at a pressure of 1.05 bar.

The isentropic efficiency of the compressor is 0.84.

The isentropic efficiency of LP turbine is 0.86.

The mass flow of fuel and all system losses may be ignored.

(a) Sketch the cycle on a Temperature-specific entropy diagram. (3)

(b) Calculate EACH of the following:

- **(i) the compressor specific work input; (4)**
- **(ii) the specific net work output; (6)**
- **(iii) the cycle thermal efficiency. (3)**

Note: for air $\gamma = 1.4$, $c_p = 1.005$ kJ/kgK

for the combustion products $\gamma = 1.33$, $c_p = 1.15$ kJ/kgK

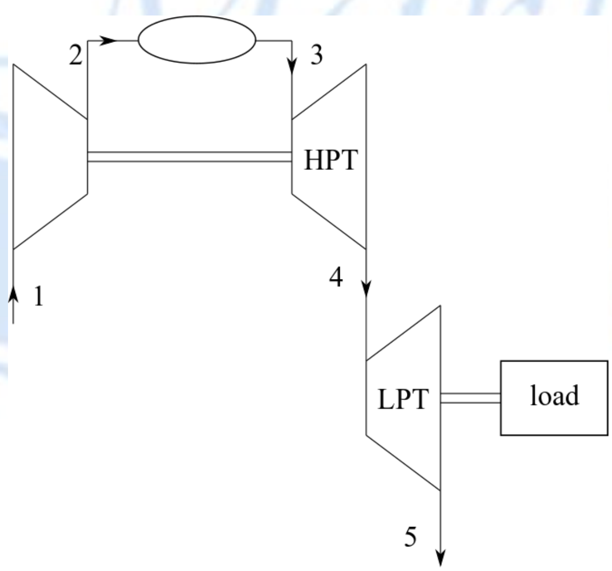
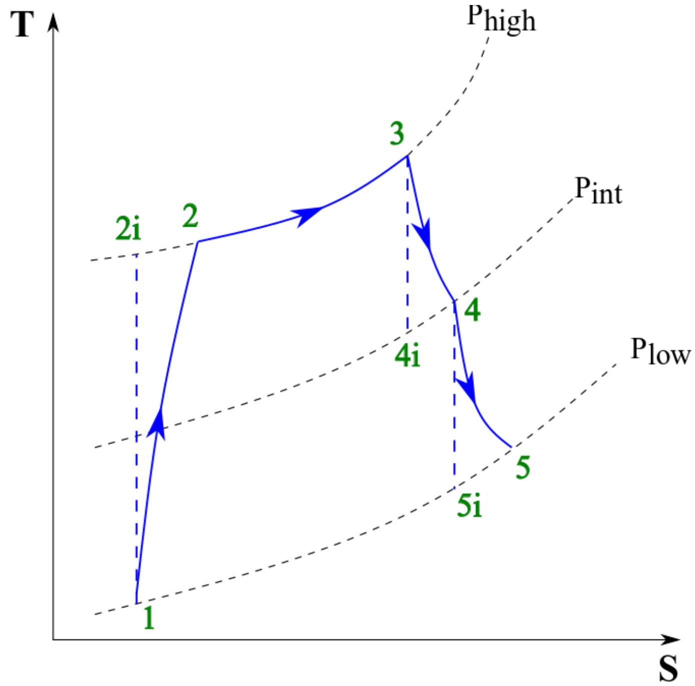


Fig Q2

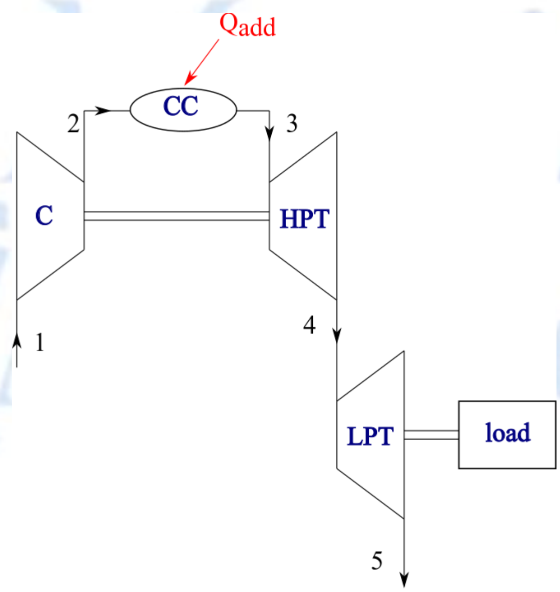
Solution:

(a)

P.T.O.



(b)



$P_{low} = P_1 = P_5 = 1.05\ bar = 105\ kN/m^2$

$T_1 = 15\ ^oC = 288\ K$

$P_{high} = P_2 = P_3 = 9.45\ bar = 945\ kN/m^2$

$T_3 = 1027^oC = 1300\ K$

$P_{int} = P_4 = 3.23\ bar = 323\ kN/m^2$

$\eta_C = 0.84, \quad \eta_{LPT} = 0.86$

$\gamma_{air} = 1.4, \quad c_{p\ air} = 1.005\ kJ/kgK$

$\gamma_{gas} = 1.33, \quad c_{p\ gas} = 1.15\ kJ/kgK$

$$T_{2i} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 288 \times \left(\frac{945}{105}\right)^{\frac{0.4}{1.4}} = 539.55\ K$$

$$\eta_C = \frac{T_{2i} - T_1}{T_2 - T_1}$$

$$\Rightarrow\ T_2 = \frac{T_{2i} - T_1}{\eta_C} + T_1 = \frac{539.55 - 288}{0.84} + 288 = 587.464\ K$$

(b)(i)

$$W_c = c_{p\,air}(T_2 - T_1) = 1.005 \times (587.464 - 288) = 300.96\ kJ/kg$$

Therefore, the compressor specific work input is 300.96 kJ/kg.

(b)(ii)

$$W_c = W_{HPT}$$

$$\Rightarrow\ 300.96 = c_{p\,gas}(T_3 - T_4)$$

$$\Rightarrow\ 300.96 = 1.15(1300 - T_4)$$

$$\Rightarrow\ T_4 = 1300 - \frac{300.96}{1.15} = 1038.295\ K$$

$$T_{5i} = T_4\left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = 1038.295 \times \left(\frac{105}{323}\right)^{\frac{0.33}{1.33}} = 785.66\ K$$

$$\eta_{LPT} = \frac{T_4 - T_5}{T_4 - T_{5i}}$$

$$\Rightarrow\ \ T_5 = T_4 - \eta_{LPT}(T_4 - T_{5i}) = 1038.295 - 0.86(1038.295 - 785.66) = 821.03\ K$$

$$W_{net,out} = W_{LPT} + W_{HPT} - W_C = W_{LPT} = c_{p\,gas}(T_4 - T_5)$$

$$= 1.15(1038.295 - 821.03) = 249.85\ kJ/kg$$

Therefore, the specific net work output is 249.85 kJ/kg.

(b)(iii)

$$\eta_{th} = \frac{W_{net,out}}{Q_{add}} = \frac{W_{net,out}}{c_{p\,gas}(T_3 - T_2)} = \frac{249.85}{1.15(1300 - 587.464)} = 0.3049 = 30.49\%$$

Therefore, the cycle thermal efficiency is 30.49%.

Q.5

In a 50% reaction turbine stage, dry saturated steam at pressure of 3 bar leaves the fixed blades with a velocity of 120 m/s.

The moving blade exit angle is 25^o and the mean blade height is 40 mm.

The axial velocity of steam is 75% of the blade velocity at the mean blade radius.

The mass flow of steam through the stage is 7200 kg/hr.

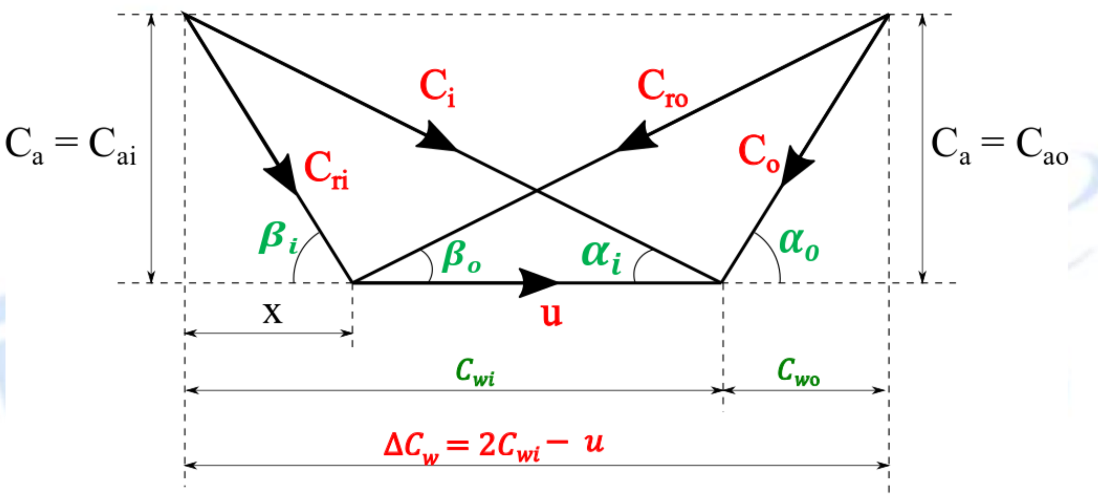
(a) Sketch the stage velocity diagram indicating the velocities and relationships. (4)

(b) Calculate EACH of the following:

- **(i) the rotor speed in rev/min; (6)**
- **(ii) the diagram power; (3)**
- **(iii) the stage enthalpy drop. (3)**

Solution:

(a)



(b)

$P = 3\ bar, \qquad x = 1$

$C_i = 120\ m/s, \qquad \beta_o = 25^o, \qquad h = 40\ mm = 0.04\ m, \qquad C_a = 0.75 \times u,$

$\dot{m} = 7200\ kg/hr = 2\ kg/s$

$v = v_g @\ 3\ bar = 0.6057\ m^3/kg$

Due to symmetry in combined velocity diagram, $\alpha_i = \beta_o = 25^o$

$C_a = C_i \sin\alpha_i = 120 \times \sin 25 = 50.714\ m/s$

$C_{wi} = C_i \cos\alpha_i = 120 \times \cos 25 = 108.76\ m/s$

$C_a = 0.75 \times u \qquad \Rightarrow \qquad u = \frac{C_a}{0.75} = \frac{50.714}{0.75} = 67.618\ m/s$

$\dot{m} = \frac{\pi D h C_a}{v}$

$$\Rightarrow \quad D = \frac{\dot{m}v}{\pi C_a h} = \frac{2 \times 0.6057}{\pi \times 50.714 \times 0.04} = 0.19\, m$$

(b)(i)

$$u = \frac{\pi DN}{60} \quad \Rightarrow \quad N = \frac{u \times 60}{\pi D} = \frac{67.618 \times 60}{\pi \times 0.19} = 6796.88\, rev/min$$

Therefore, the rotor speed is 6796.88 rev/min.

(b)(ii)

$$Diagram\ power = \dot{m} \times u \times \Delta C_w = \dot{m} \times u \times (2C_{wi} - u)$$

$$= 2 \times 67.618 \times (2 \times 108.76 - 67.618) = 20272.14\, W$$

Therefore, the diagram power is 20272.14 W.

(b)(iii)

$$x = C_{wi} - u = 108.76 - 67.618 = 41.142\, m/s$$

$$C_o = C_{ri} = \sqrt{x^2 + {C_a}^2} = \sqrt{41.142^2 + 50.714^2} = 65.3\, m/s$$

$$C_{ro} = C_i = 120\, m/s$$

$$Stage\ enthalpy\ drop = \frac{{C_i}^2 - {C_o}^2}{2} + \frac{{C_{ro}}^2 - {C_{ri}}^2}{2} = \frac{120^2 - 65.3^2}{2} + \frac{120^2 - 65.3^2}{2}$$

$$= 10135.91\, J/kg$$

Therefore, the stage enthalpy drop is 10135.91 J/kg.

Q.6

A vapour compression refrigeration plant uses R134a.
The refrigerant enters the compressor at a pressure and temperature of 1.6393 bar and -5°C respectively and undergoes isentropic compression to 13.174 bar.
The liquid refrigerant leaves the condenser at a temperature of 35°C.
The cooling load is 250 kW.

(a) Sketch the cycle on a pressure-specific enthalpy diagram indicating areas of heat and work transfer. (2)

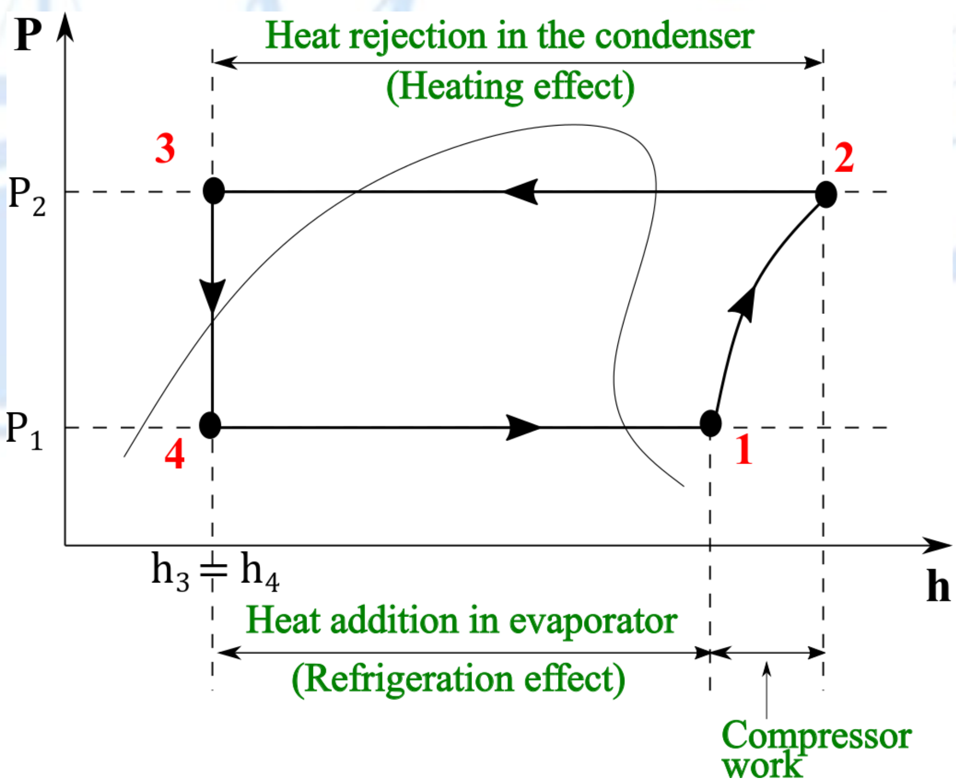
(b) Sketch the cycle on a Temperature-specific entropy diagram indicating areas of superheat and sub cooling. (2)

(c) Calculate EACH of the following:

- **(i) the mass of dry saturated vapour entering the evaporator; (4)**
- **(ii) the compressor power; (4)**
- **(iii) the coefficient of performance; (2)**
- **(iv) the Carnot coefficient of performance between the same temperature limits. (2)**

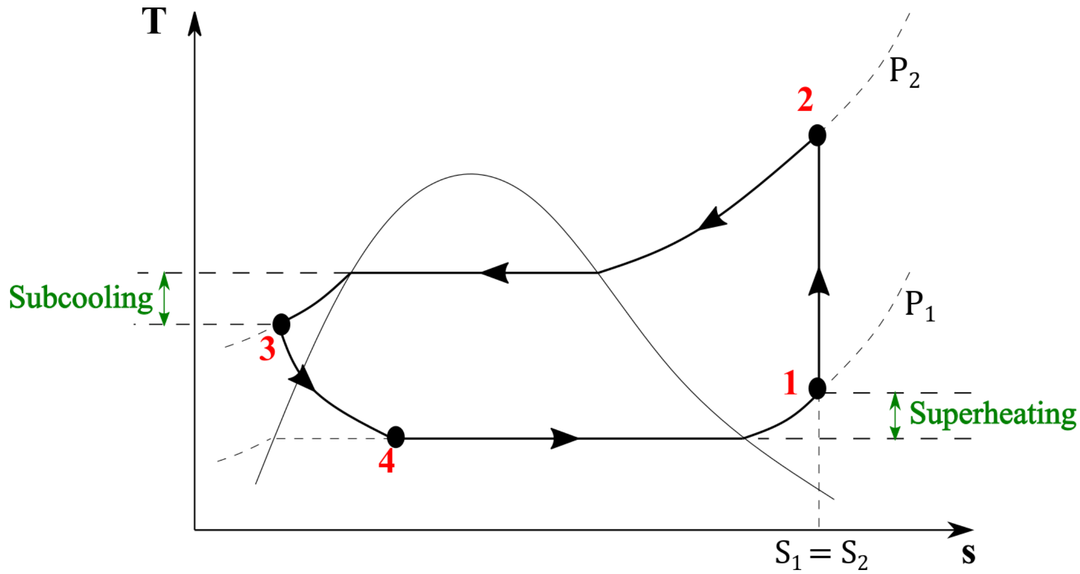
Solution:

(a)



PTO

(b)



(c)

$Refrigerant = R134a$

$P_1 = 1.6393\ bar, \quad T_1 = -5^oC, \quad P_2 = 13.174\ bar$

$s_1 = s_2 \quad \{\because Isentropic\ compression\}$

$Cooling\ load = 250\ kW$

$T_{sat}@P_1 = -15^oC, \quad T_{sat}@P_2 = 50^oC$

$(Superheat)_1 = T_1 - T_{sat}@P_1 = -5 - (-15) = 10\ K$

$h_1 = h@P_1\ \&\ 10K\ superheat = 397.86\ kJ/kg$

$s_1 = s@P_1\ \&\ 10K\ superheat = 1.7683\ kJ/kgK$

$s_2 = s_1 = 1.7683\ kJ/kgK$

$At\ P_2 = 13.174\ bar$

s (kJ/kgK)	h (kJ)
1.7438	435.44
1.7683	h_2
1.7775	446.84

$$h_2 = 435.44 + (1.7683 - 1.7438) \times \frac{446.84 - 435.44}{1.7775 - 1.7438} = 443.728\ kJ/kg$$

$h_4 = h_3 = h_f@T_3 = 248.98\ kJ/kg$

$$Cooling\ load = \dot{m}(h_1 - h_4)$$

$$\Rightarrow \quad \dot{m} = \frac{Cooling\ load}{(h_1 - h_4)} = \frac{250}{397.86 - 248.98} = 1.679\ kg/s$$

$$h_f@P_1 = 180.16\ kJ/kg, \qquad h_g@P_1 = 389.49\ kJ/kg$$

$$x_4 = \frac{h_4 - h_f}{h_g - h_f} = \frac{248.98 - 180.16}{389.49 - 180.16} = 0.3287$$

(c)(i)

$$Mass\ of\ dry\ saturated\ vapour\ entering\ the\ evaporator = x_4 \times \dot{m}$$

$$= 0.3287 \times 1.679 = 0.5518\ kg/s$$

Therefore, the mass of dry saturated vapour entering the evaporator is 0.5518 kg/s.

(c)(ii)

$$Compressor\ power = \dot{m}(h_2 - h_1) = 1.679(443.728 - 397.86) = 77.0123\ kW$$

Therefore, the compressor power is 77.0123 kW.

(c)(iii)

$$COP = \frac{Cooling\ load}{Compressor\ power} = \frac{250}{77.0123} = 3.246$$

Therefore, the coefficient of performance is 3.246.

(c)(iv)

$$T_L = -15^oC = 258\ K$$

$$T_H = 50^oC = 323\ K$$

$$COP_{Carnot} = \frac{T_L}{T_H - T_L} = \frac{258}{323 - 258} = 3.97$$

Therefore, the Carnot coefficient of performance is 3.97.

Q.7

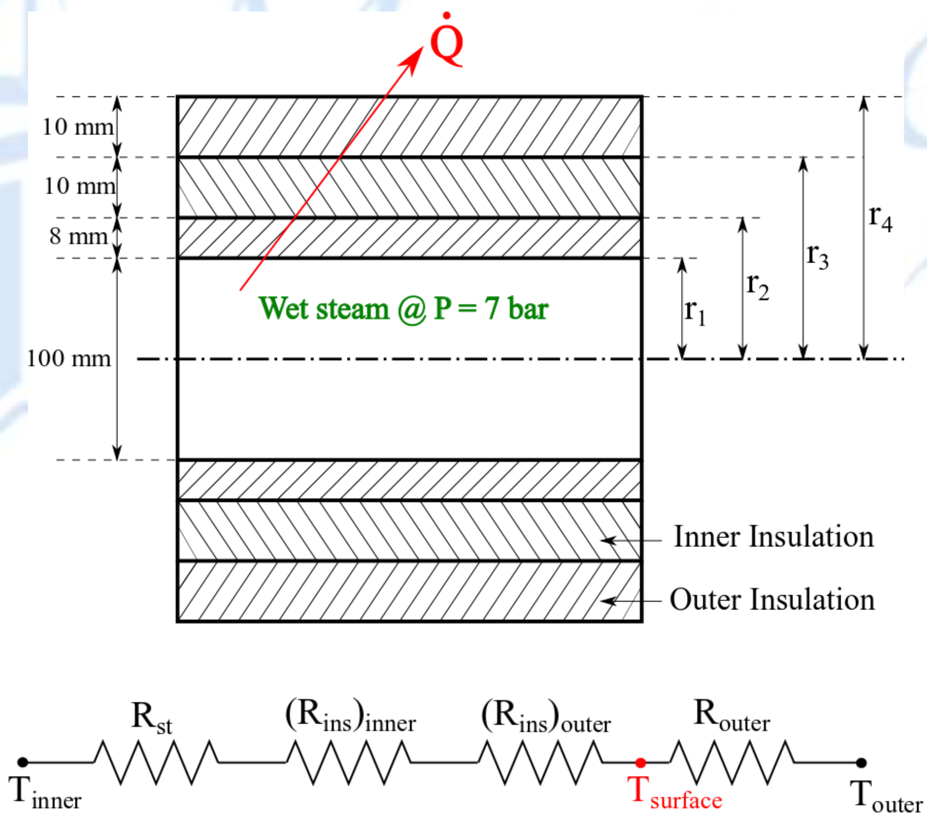
A steel pipe has an internal diameter of 100 mm and a wall thickness of 8 mm.
It carries wet steam a pressure of 7 bar and is carried with two layers of insulation, each 10 mm thick.
The outer layer of insulation becomes contaminated and is removed.
The surrounding air temperature remains constant at 20°C.
Calculate EACH of the following:

(a) the heat loss per metre length of pipe when covered in two layers of insulation; (6)
(b) the percentage increase in heat loss when the outer layer is removed; (5)
(c) the percentage increase in outer surface temperature. (5)

Note: the heat transfer coefficient of the inner surface may be ignored.
the thermal conductivity of steel = 52 W/mK
the thermal conductivity of the inner insulation = 0.045 W/mK
the thermal conductivity of the outer insulation = 0.13 W/mK
the heat transfer coefficient of the outer surface = 15 W/m²K

Solution:

(a)



$r_1 = 50\ mm = 0.05\ m$

$r_2 = 50 + 8 = 58\ mm = 0.058\ m$

$r_3 = 58 + 10 = 68\ mm = 0.068\ m$

$r_4 = 68 + 10 = 78\ mm = 0.078\ m$

Wet steam in pipe at $P = 7\ bar$

$\therefore T_{inner} = T_{sat}@7bar = 165^oC$

$T_{outer} = 20^oC$

$k_{st} = 52\ W/mK, \qquad (k_{ins})_{inner} = 0.045\ W/mK, \qquad (k_{ins})_{outer} = 0.13\ W/mK$

$h_{outer} = 15\ W/m^2K, \quad l = 1\ m$

$$R_{st} = \frac{1}{2\pi k_{st} l}\ln\frac{r_2}{r_1} = \frac{1}{2\pi \times 52 \times 1} \times \ln\frac{0.058}{0.05} = 4.542\ \times 10^{-4}\ K/W$$

$$(R_{ins})_{inner} = \frac{1}{2\pi (k_{ins})_{inner} l}\ln\frac{r_3}{r_2} = \frac{1}{2\pi \times 0.045 \times 1} \times \ln\frac{0.068}{0.058} = 0.5625\ K/W$$

$$(R_{ins})_{outer} = \frac{1}{2\pi (k_{ins})_{outer} l}\ln\frac{r_4}{r_3} = \frac{1}{2\pi \times 0.13 \times 1} \times \ln\frac{0.078}{0.068} = 0.1679\ K/W$$

$$R_{outer} = \frac{1}{h_{outer} A_{outer}} = \frac{1}{15 \times 2\pi \times 0.078 \times 1} = 0.136\ K/W$$

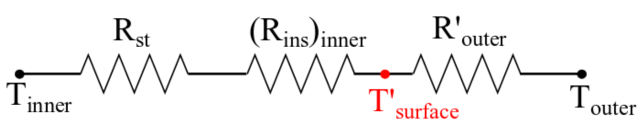
$$\dot{Q} = \frac{T_{inner} - T_{outer}}{R_{st} + (R_{ins})_{inner} + (R_{ins})_{outer} + R_{outer}}$$

$$= \frac{165 - 20}{4.542\ \times 10^{-4} + 0.5625 + 0.1679 + 0.136} = 167.27\ W$$

Therefore, the heat loss per metre length of pipe when covered in two layers of insulation is 167.27 W.

(b)

After removing outer insulation



$$R'_{outer} = \frac{1}{h_{outer} \times A_{outer}} = \frac{1}{15 \times 2\pi \times 0.068 \times 1} = 0.156\ W$$

P.T.O.

Rate of heat loss without outer insulation

$$\dot{Q}_{without} = \frac{T_{inner} - T_{outer}}{R_{st} + (R_{ins})_{inner} + R'_{outer}} = \frac{165 - 20}{4.542 \times 10^{-4} + 0.5625 + 0.156}$$

$$= 201.68\ W$$

$$Percentage\ increase\ in\ heat\ loss = \frac{\dot{Q}_{without} - \dot{Q}}{\dot{Q}} \times 100 = \frac{201.68 - 167.27}{167.27} \times 100$$

$$= 20.57\%$$

Therefore, the percentage increase in heat loss when the outer layer is removed is 20.57%.

(c)

With outer insulation

$$\dot{Q} = \frac{T_{surface} - T_{outer}}{R_{outer}}$$

$$\Rightarrow T_{surface} = \dot{Q} \times R_{outer} + T_{outer} = 167.27 \times 0.136 + 20 = 42.74^{o}C$$

(surface temperature with insulation)

Without outer insulation

$$\dot{Q}_{without} = \frac{T'_{surface} - T_{outer}}{R'_{outer}}$$

$$\Rightarrow T'_{surface} = \dot{Q}_{without} \times R'_{outer} + T_{outer} = 201.68 \times 0.156 + 20 = 51.46^{o}C$$

(Surface temperature without outer insulation)

$$\%\ increase\ in\ outer\ surface\ temperature = \frac{T'_{surface} - T_{surface}}{T_{surface}} \times 100$$

$$= \frac{51.46 - 42.74}{42.74} \times 100 = 20.40\%$$

Therefore, the percentage increase in outer surface temperature is 20.40%.

Q.8

A single acting, two stage reciprocating compressor is designed for minimum work with perfect intercooling.

The low pressure cylinder contains a mass of 0.02 kg of air when the piston is at bottom dead centre.

The air is compressed from a pressure and temperature of 0.95 bar and 25°C respectively through an overall pressure ratio of 16:1.

The index of expansion and compression in both stages is 1.28.

The clearance volume in each stage is 5% of the respective swept volume and the compressor runs at a speed of 500 rev/min.

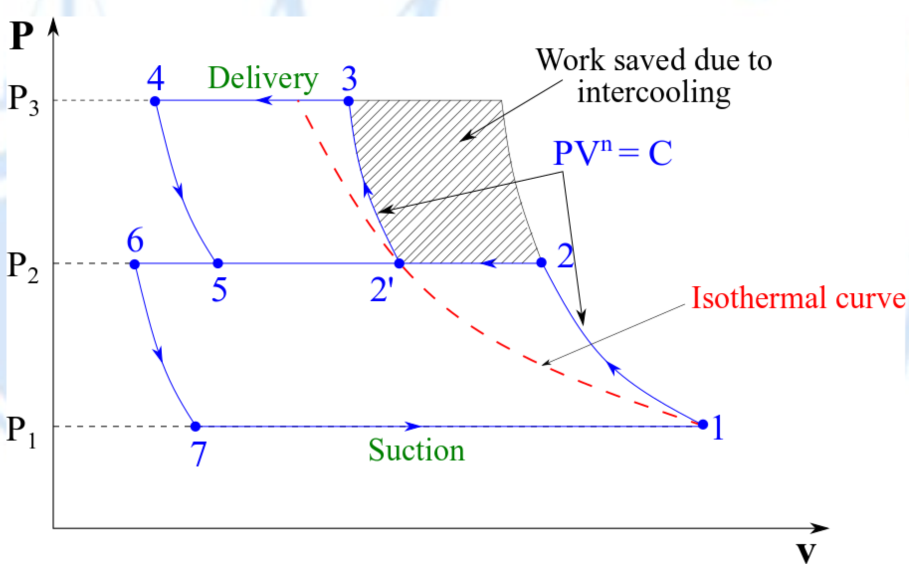
(a) Sketch the cycle on a pressure-volume diagram. (2)

(b) Calculate EACH of the following:

- **(i) the compressor indicated power; (6)**
- **(ii) the heat rejected during the compression process; (4)**
- **(iii) the rate of heat rejection in the intercooler. (4)**

Note: for air $\gamma = 1.4$, $c_p = 1.005$ kJ/kgK and $R = 0.287$ kJ/kgK

(a)



(b)

$$m_1 = 0.02\ kg$$

$$P_1 = 0.95\ bar = 95\ kN/m^2$$

$$T_1 = 25^oC = 298\ K$$

$$\frac{P_3}{P_1} = 16$$

$$n = 1.28$$

$$V_c = 0.05V_s \quad \Rightarrow \quad \frac{V_c}{V_s} = 0.05$$

$N = 500\ rev/min$

$\gamma = 1.4, \quad c_p = 1.005\ kJ/kgK, \quad R = 0.287\ kJ/kgK$

$c_v = c_p - R = 1.005 - 0.287 = 0.718\ kJ/kgK$

$$P_1V_1 = m_1RT_1 \quad \Rightarrow \quad V_1 = \frac{m_1RT_1}{P_1} = \frac{0.02 \times 0.287 \times 298}{95} = 0.018\ m^3$$

$$V_1 = V_s + V_c = V_s + 0.05V_s = 1.05V_s$$

$$\Rightarrow V_s = \frac{V_1}{1.05} = \frac{0.018}{1.05} = 0.0171\ m^3$$

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \left(\frac{P_3}{P_1}\right)^{\frac{1}{2}} = 16^{\frac{1}{2}} = 4 \qquad \{\because Minimum\ work\ input$$

$$\eta_{vol} = 1 + \frac{V_c}{V_s}\left[1 - \left(\frac{P_2}{P_1}\right)^{1/n}\right] = 1 + 0.05\left[1 - 4^{1/1.28}\right] = 0.9023$$

$$\eta_{vol} = \frac{\dot{m}}{\frac{P_1}{RT_1} \times V_s \times \frac{N}{60}}$$

$$\Rightarrow \quad \dot{m} = \eta_{vol} \times \frac{P_1}{RT_1} \times V_s \times \frac{N}{60} = 0.9023 \times \frac{95}{0.287 \times 298} \times 0.0171 \times \frac{500}{60}$$

$$= 0.1428\ kg/s$$

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{0.28}{1.28}} = 298 \times 4^{0.28/1.28} = 403.568\ K$$

(b)(i)

$$IP = 2 \times \frac{n}{n-1} \times \dot{m}R(T_2 - T_1) = 2 \times \frac{1.28}{0.28} \times 0.1428 \times 0.287 \times (403.568 - 298)$$

$$= 39.557\ kW$$

Therefore, the compressor indicated power is 39.557 kW.

P.T.O.

(b)(ii)

From 1st law relation

$Q_{1-2} - W_{1-2} = \Delta U_{1-2}$

$$\Rightarrow \quad Q_{1-2} = \Delta U_{1-2} + W_{1-2} = m_1 c_v (T_2 - T_1) + \frac{m_1 R (T_1 - T_2)}{n-1}$$

$$= 0.02 \times 0.718 \times (403.568 - 298) + \frac{0.02 \times 0.287 \times (298 - 403.568)}{0.28}$$

$$= -0.6482\ kJ = 0.6482\ kJ\ (Heat\ rejction)$$

$Total\ heat\ rejection\ during\ compression = 2 \times Q_{1-2} = 2 \times 0.6482 = 1.2964\ kJ$

Therefore, the heat rejected during compression process is 1.2964 kJ.

(b)(iii)

$\dot{Q}_{ic} = \dot{m} c_p (T_2 - T_{2'})$

$\Rightarrow \quad \dot{Q}_{ic} = \dot{m} c_p (T_2 - T_1) \qquad \{\because Perfect\ intercooling \ \Rightarrow \ T_{2'} = T_1$

$\Rightarrow \quad \dot{Q}_{ic} = 0.1428 \times 1.005 \times (403.568 - 298) = 15.15\ kW$

Therefore, the rate of heat rejection in the intercooler is 15.15 kW.

Q.9

A straight section of horizontal pipe tapers in diameter from 300 mm at inlet to 150 mm at outlet.

The mass flow of fresh water through the pipe is 500 tonne per hour and the pressure at inlet is 3 bar.

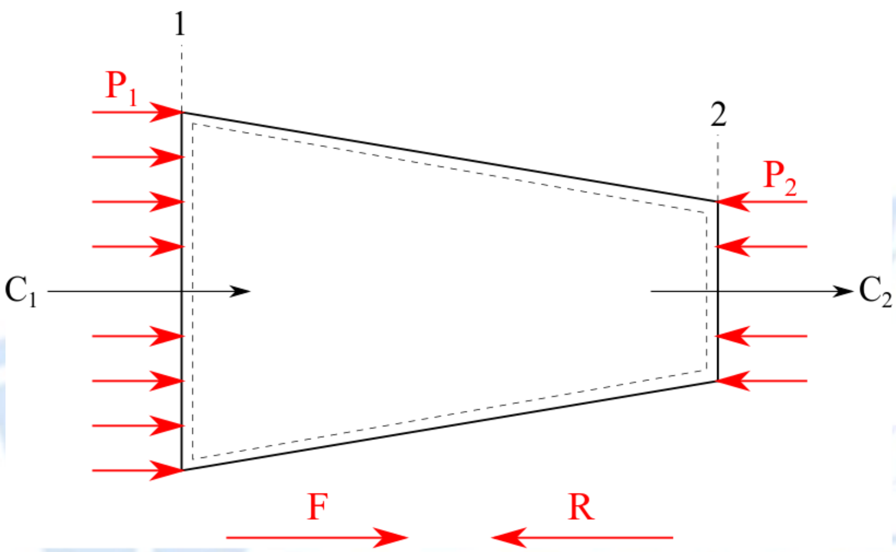
(a) Calculate EACH of the following:

- **(i) the fluid velocity at the outlet end; (2)**
- **(ii) the pressure at the outlet end; (5)**
- **(iii) the longitudinal thrust on the pipe. (5)**

(b) Sketch a diagram of forces acting on the pipe. (4)

Solution:

(b)



Where,

$R = Force\ on\ water\ due\ to\ pipe$

$F = Longitudial\ thrust\ on\ the\ pipe\ or\ force\ on\ the\ pipe\ due\ water\ flow$

Here, F and R equal in magnitude and opposite in direction

(a)

$z_1 = z_2 \quad \{\because Horizontal\ pipe$

$D_1 = 300\ mm = 0.3\ m, \quad D_2 = 150\ mm = 0.15\ m$

$\dot{m} = 500\ tonne/hour = 500 \times \frac{1000}{3600}\ kg/s = 138.889\ kg/s$

$\rho = 1000\ kg/m^3 \quad \{\because Fresh\ water\ flowing\ through\ the\ pipe$

$P_1 = 3\ bar = 3 \times 10^5\ N/m^2$

(a)(i)

$$\dot{m} = A_2 C_2 \rho$$

$$\Rightarrow \quad C_2 = \frac{\dot{m}}{A_2 \rho} = \frac{138.889}{\frac{\pi}{4} \times 0.15^2 \times 1000} = 7.86\, m/s$$

Therefore, the fluid velocity at the outlet end is 7.86 m/s.

(a)(ii)

$$\dot{m} = A_1 C_1 \rho$$

$$\Rightarrow \quad C_1 = \frac{\dot{m}}{A_1 \rho} = \frac{138.889}{\frac{\pi}{4} \times 0.3^2 \times 1000} = 1.965\, m/s$$

From Bernoulli equation,

$$\frac{P_1}{\rho g} + \frac{{C_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{C_2}^2}{2g} + z_2$$

$$\Rightarrow \quad \frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{{C_1}^2}{2g} - \frac{{C_2}^2}{2g} \qquad \{\because\ z_1 = z_2$$

$$\Rightarrow \quad P_2 = P_1 + \rho \times \frac{{C_1}^2 - {C_2}^2}{2} = 3 \times 10^5 + 1000 \times \frac{1.965^2 - 7.86^2}{2}$$

$$\Rightarrow \quad P_2 = 271040.80\, N/m^2 = 2.71\, bar$$

Therefore, the pressure at the outlet end is 2.71 bar.

(a)(iii)

From momentum equation,

$$\Sigma F = \dot{m}(C_2 - C_1)$$

$$\Rightarrow \quad P_1 A_1 - P_2 A_2 - R = \dot{m}(C_2 - C_1)$$

$$\Rightarrow \quad R = P_1 A_1 - P_2 A_2 - \dot{m}(C_2 - C_1)$$

$$\Rightarrow \quad R = 3 \times 10^5 \times \frac{\pi}{4} \times 0.3^2 - 271040.80 \times \frac{\pi}{4} \times 0.15^2 - 138.889 \times (7.86 - 1.965)$$

$$\Rightarrow \ R = 15597.31\, N$$

$$\therefore\ F = R = 15597.31\, N$$

Therefore, the longitudinal thrust on the pipe is 15597.31 N.